FULLY WORKED SOLUTIONS

Context 7: The physics of music

Chapter 17: Sound thinking

Chapter questions

1. $v = 3900\ \text{m s}^{-1}$, $\lambda = 6$ m

   $f = v/\lambda = 3900/6 = 650$ Hz

2. $f = 512$ Hz

   (a) The frequency will be the same, so $f = 512$ Hz

   (b) $v = 5170\ \text{m s}^{-1}$, $f = 512$ Hz

   $\lambda = v/f = 5170/512 = 10.1$ m

3. (a) $T = -5$ °C

$$v = 331\sqrt{\frac{-5}{273} + 1} = 328\ \text{m s}^{-1}$$

   (b) $T = 40$ °C

$$v = 331\sqrt{\frac{40}{273} + 1} = 354\ \text{m s}^{-1}$$

4. $$I \propto \frac{1}{d^2}$$

$$\frac{I_2}{I_1} = \frac{d_1^2}{d_2^2}$$

   $I_1 = 4\ \text{W m}^{-2}$, $I_2 = 1\ \text{W m}^{-2}$, $d_1 = 2$ m

$$\frac{1}{4} = \frac{2^2}{d_2^2}$$

   $d_2^2 = 16$

   $d_2 = 4$ m

5. $I = P/A$

   $I = E/tA$

   $E/A = It = 0.2 \times 10^{-3} \times 20 = 4 \times 10^{-3}\ \text{J m}^{-2}$

6. $P = I \times 4\pi d^2 = 2 \times 10^{-3} \times 4\ \pi \times 3^2$

   $P = 0.23$ W

7. $L = 10 \log I/I_o$

   $20 = 10 \log I/10^{-12}$

   $2 = \log I/10^{-12}$

$10^2 = I/10^{-12}$

$I = 10^{-10}$ W m^{-2}

8. $L = 10 \log I/I_o$

$I/I_o = 10^{L/10}$

$I = 10^{L/10} \times 10^{-12} = 10^{L/10 - 12}$

For a scream: $L = 120$ dB

$I_{scream} = 10^{(120/10) - 12} = 10° = 1$ W m^{-2}

For normal conversation: $L = 60$ dB

$I_{conversation} = 10^{(60/10) - 12} = 10^{-6}$ W m^{-2}

$$\frac{I_{scream}}{I_{conversation}} = \frac{1}{10^{-6}} = 10^6$$

9. $L = 10 \log I/I_o$

$L = 10 \log (1000\, I_o/I_o)$

$= 10 \log 1000$

$= 30$ dB

Review questions

6. Values obtained from the diagram:

(a) ~ 6000 Hz

(b) ~ 78 dB

(c) below 20 Hz

(d) ~ 1400 Hz to 7 500 Hz

(e) ~ 45 Hz to 100 Hz, 8000 Hz to 15 000 Hz

7. $\lambda = 1.5$ m, $d_1 = 4.5$ m

At a minimum position, the difference between path lengths Δd must be a whole number multiple of $\frac{\lambda}{2}$. In this case, where the distance from the second speaker must be at least that from the first speaker:

$\Delta d = d_2 - d_1 = \frac{n\lambda}{2}$ where $n = 1, 2, 3\ldots$

The smallest value of n for which this occurs is $n = 1$.

Therefore,

$d_2 - 4.5 = \frac{1.5}{2} = 0.75$ m

$d_2 = 0.75 + 4.5 = 5.25$ m

8. $I = 10^{-8}$ W m^{-2}, $I_o = 10^{-12}$ W m^{-2}

$$L = 10 \log \left(\frac{I}{I_o}\right) = 10 \log\left(\frac{10^{-8}}{10^{-12}}\right)$$

$= 40$ dB

9. $L = 65$ dB, $I_o = 10^{-12}$ W m^{-2}

$$L = 10 \log\left(\frac{I}{I_o}\right)$$

$$65 = 10 \log \left(\frac{I}{10^{-12}}\right)$$

$$6.5 = \log \left(\frac{I}{10^{-12}}\right)$$

$$10^{6.5} = \left(\frac{I}{10^{-12}}\right)$$

$$I = 10^{6.5} \times 10^{-12}$$

$$I = 3.2 \times 10^{-6} \text{ W m}^{-2}$$

10. $L = 100$ dB, $I_o = 10^{-12}$ W m^{-2}

$$100 = 10 \log \left(\frac{I}{10^{-12}}\right)$$

$$10 = \log \left(\frac{I}{10^{-12}}\right)$$

$$10^{10} = \left(\frac{I}{10^{-12}}\right)$$

$$I = 10^{10} \times 10^{-12} = 10^{-2} \text{ W m}^{-2}$$

$$P = 4\pi d^2 I = 4\pi \times 10^2 \times 10^{-2}$$

$$P = 12.6 \text{ W}$$

$$\text{Percentage converted} = \frac{12.6}{20} \times 100\% = 63\ \%$$

11. (a) $T = 30$ °C

$$v_T = 331 \sqrt{\frac{T}{273} + 1}$$

$$= 331 \sqrt{\frac{30}{273} + 1}$$

$$= 349 \text{ m s}^{-1}$$

(b) $d = v_T t = 349 \times 7 = 2.4$ km

12. Total distance travelled $= 2d$

$t = 0.17$ s, $v = 1\ 540$ m s^{-1}

$2d = vt = 1\ 540 \times 0.17 = 261.8$ m

$d = 131$ m

13. $v_{20} = 344$ m s^{-1}, $\lambda = 0.800$ m

$$f = \frac{v}{\lambda} = \frac{344}{0.8} = 430\ Hz$$

(a) $f = 430$ Hz, $v_{iron} = 5\ 120$ m s^{-1}

$$\lambda = \frac{v}{f} = \frac{5120}{430} = 11.9\ \text{m}$$

(b) $f = 430$ Hz, $v_{seawater} = 1\ 540$ m s^{-1}

$$\lambda = \frac{v}{f} = \frac{1540}{430} = 3.6\ \text{m}$$

(c) $f = 430$ Hz, $v_{pure\ water} = 1\ 498$ m s^{-1}

$$\lambda = \frac{v}{f} = \frac{1498}{430} = 3.5\ \text{m}$$

14. distance = $2d$, $t = 2.8$ s, $v = 344$ m s^{-1}

$2d = vt = 344 \times 2.8 = 963$ m

$d = 481.6$ m ≈ 482 m

15. (a) $T = 1.5$ ms $= 1.5 \times 10^{-3}$ s

(b) $f = 1/T = \dfrac{1}{1.5 \times 10^{-3}} = 667$ Hz

(c) $$\lambda = \frac{v}{f} = \frac{344}{667} = 0.52\ \text{m}$$

16. As $I \propto \dfrac{1}{A}$, $\dfrac{I_{eardrum}}{I_{eartrumpet}} = \dfrac{A_{eartrumpet}}{A_{eardrum}}$

$A_{eartrumpet} = \pi \times (7.5)^2 = 177$ cm^2

$A_{eardrum} = 0.5$ cm^2

$$\frac{I_{eardrum}}{I_{eartrumpet}} = \frac{A_{eartrumpet}}{A_{eardrum}} = \frac{177}{0.5} = 88.5$$

The ear trumpet increases the intensity of the sound by a factor of 88.5.

17. $v = \dfrac{d}{t} = \dfrac{400}{12} = 333$ m s^{-1}

$$v_T = 331\sqrt{\frac{T}{273} + 1}$$

$$333 = 331\sqrt{\frac{T}{273} + 1}$$

$$1.014 = \frac{T}{273} + 1$$

$$T = (1.014 - 1)\ 273 = 3.9\ ^{\circ}\text{C}$$

18. $T_k = T_c + 273$

$T_1 = 18 + 273 = 291$ K

$T_2 = 0 + 273 = 273$ K

$v_2 = 1\ 284$ m s^{-1}

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$$

$$v_1 = v_2 \sqrt{\frac{T_1}{T_2}}$$

$$= 1284 \sqrt{\frac{291}{273}}$$

$$v_1 = 1326 \text{ m s}^{-1}$$

20. $L_{Al} = L_{Cu}$

$v_{Al} = 5\ 100$m s^{-1}, $v_{Cu} = 3\ 900$ m s^{-1}

$L = vt$ for both metals, so

$L_{Al} = v_{Al}t_{Al}$ and $L_{Cu} = v_{Cu}t_{Cu}$

As $L_{Al} = L_{Cu}$,

$v_{Al}t_{Al} = v_{Cu}t_{Cu}$

Also, as $t_{Cu} = t_{Al} + 0.0003$

$v_{Al}t_{Al} = v_{Cu}(t_{Al} + 0.0003)$

$5100t_{Al} = 3900t_{Al} + 1.17$

$1200t_{Al} = 1.17$

$t_{Al} = 0.975$ ms

$L = v_{Al}t_{Al} = 5\ 100 \times 9.75 \times 10^{-4} = 4.97 \text{ m} \approx 5 \text{ m}$

22. (a)

***T* (°C)**	$v_T = 331\sqrt{\frac{T}{273} + 1}$ **(m s^{-1})**	$v_T = 331 + 0.60\ T$ **(m s^{-1})**
–50	299	301
–30	312	313
–10	325	325
0	331	331
10	337	337
30	349	349
50	360	361

70	371	373

The equations give the same values for v_T in the range between –10 °C and 30 °C. At the extremes of the range, they differ only to within 0.7%.